Probability-One Homotopy for Robust Initialization of Differential-Algebraic Equations

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An evolution of the recently introduced operator `homotopy()` is proposed, which further improves the solution of difficult initialization problems. The background and motivation for this approach are discussed and it is demonstrated how to apply it for electrical and fluid systems. The key difference to the earlier approach is the supporting theory, which guarantees that the method converges globally with probability one.

Summary

Previously [3], we introduced a homotopy operator. It maps `homotopy(actual = ..., simplified = ...)` to $\lambda \cdot actual + (1-\lambda) \cdot simplified$. The advantage of this approach is that the concept is simple and easy to understand. Also, it was successfully tested on relevant test cases [3, 1]. It has certain limitations however, in particular that the homotopy map is hard-wired into the language specification, that convergence is based on heuristics, and that a naive application can lead to singular problems (e.g., with a singular Jacobian at $\lambda = 0$).

The objective of this contribution is thus to propose a more powerful homotopy operator, which can be used as the original one, enables a declarative definition of arbitrary homotopy maps, and allows global convergence via probability-one homotopy [2, 4], an established method from topology.

Informally, the key elements of probability-one homotopy are a well-defined random element to guarantee the full rank of the Jacobian matrix, a boundedness argument, and an embedding, which essentially corresponds to the simplifications of component governing equations applied in [3, 1].

In the paper, a key theorem is reviewed to summarize the supporting theory. Additionally, implementation aspects in Modelica tools are discussed and a number of application examples are given.

References

[1] F. Casella, L. Savoldelli, and M. Sielemann. Steady-state initialization of object-oriented thermo-fluid models by homotopy methods. In *Proceedings of Eighth International Modelica Conference*, Dresden, Germany, March 2011.

[2] S.-N. Chow, J. Mallet-Paret, and J. A. Yorke. Finding zeroes of maps: Homotopy methods that are constructive with probability one. *Math. Comput.*, 32(143):887–899, July 1978.

[3] M. Sielemann, F. Casella, M. Otter, C. Clauss, J. Eborn, S. Mattsson, and H. Olsson. Robust initialization of differential-algebraic equations using homotopy. In *Proceedings of Eighth International Modelica Conference*, Dresden, Germany, March 2011.

[4] L. T. Watson. Globally convergent homotopy methods: A tutorial. *Appl. Math. Comput.*, 31:369–396, May 1989.